NCPC 2017
Presentation of solutions

The Jury

2017-10-07
NCPC 2017 Jury

- Per Austrin (KTH Royal Institute of Technology)
- Pål Grønås Drange (Statoil ASA)
- Markus Fanebust Dregi (Statoil ASA/Webstep)
- Antti Laaksonen (CSES)
- Ulf Lundström (Excillum)
- Jimmy Mårdell (Spotify)
- Lukáš Poláček (Google)
- Johan Sannemo (Google)
- Pehr Söderman (Kattis)
Problem
Classify moose based on their horns.

Some solution (guess the language)
solve(0, 0) :-
    !, write('Not a moose').
solve(L, R) :-
    type(L, R, Type),
    Val is 2*max(L, R),
    write(Type), write(' '), write(Val).
type(_, _, "Even") :- !.
type(_, _, "Odd").

Statistics: 347 submissions, 252 accepted, first after 00:03
Problem

Pick best relay team, given runners’ standing and flying start times.

Solution

1. Pre-sort runners by their flying start time
Problem
Pick best relay team, given runners' standing and flying start times.

Solution
1. Pre-sort runners by their flying start time
2. Try every runner on the first leg
B — Best Relay Team

Problem
Pick best relay team, given runners’ standing and flying start times.

Solution
1. Pre-sort runners by their flying start time
2. Try every runner on the first leg
3. For every choice, fill up with 3 fastest remaining flying start runners

Complexity is $O(n \log n)$. Many other solutions are also possible.

Statistics: 491 submissions, 189 accepted, first after 00:08

Problem Author: Jimmy Mårdell
NCPC 2017 solutions
Problem
Pick best relay team, given runners’ standing and flying start times.

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Statistics: 491 submissions, 189 accepted, first after 00:08
### Problem

There are $n$ teams who solve $m$ problems in an ICPC style programming contest. After each successful submission, print the rank of your team.

### Solution

1. Maintain a set $S$: the teams whose score is better than your team’s score. Your rank is $|S| + 1$. 
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1. Maintain a set $S$: the teams whose score is better than your team’s score. Your rank is $|S| + 1$.
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The *amortized* complexity of both operations is \( O(\log n) \).
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Statistics: 578 submissions, 79 accepted, first after 00:29
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Problem

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Solution

1. Use the Floyd–Warshall all pairs shortest path algorithm with diagonals initialized to \( \infty \).
2. Afterwards diagonal entry \( d(u, u) \) gives length of shortest cycle passing through \( u \).
Problem

The dependencies form a directed graph, and the task is to find a shortest cycle in a directed graph.

Solution

1. Use the Floyd–Warshall all pairs shortest path algorithm with diagonals initialized to $\infty$.
2. Afterwards diagonal entry $d(u, u)$ gives length of shortest cycle passing through $u$.
3. Reconstruct shortest cycle using the distance matrix.

Complexity is $O(n^3)$ or $O(n(n + m))$.
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4. Alternatively, run BFS from each vertex $v$ to find shortest $v$–$v$ cycle.
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Statistics: 290 submissions, 52 accepted, first after 00:30
Emptying the Baltic

**Problem**

How much water can we drain from a point at the bottom of the sea?

**Solution**

Similar to Dijkstra's or Prim's algorithms:

1. Keep track of tentative depth of each square. Upper bound on the final water level.
2. At the start, only the drainage point has known depth.
3. In each iteration, pick the square with the lowest tentative depth and mark it final. Update tentative depth of all neighbors of the picked square.

Time complexity is $O(n \log n)$, where $n = w \cdot h$ is the size of the grid.
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1. Similar to Dijkstra’s or Prim’s algorithms:
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Statistics: 296 submissions, 55 accepted, first after 00:47
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<td>Given $n$ bit vectors of length $k$, find a bit vector whose minimum Hamming distance is maximum.</td>
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Problem

Given \( n \) bit vectors of length \( k \), find a bit vector whose minimum Hamming distance is maximum.

Solution

1. There are a total of \( 2^k \) possible bit vectors.
2. Create a graph where each node is a bit vector and there is an edge between two nodes if they differ in a single bit. (aka the \( k \)-dimensional hypercube graph)
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Given $n$ bit vectors of length $k$, find a bit vector whose minimum Hamming distance is maximum.

## Solution

1. There are a total of $2^k$ possible bit vectors.
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3. Use a single BFS with the $n$ given vectors as sources to find the node whose minimum distance is maximum.
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Time complexity is $O(n + k \cdot 2^k)$
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Time complexity is \( O(n + k \cdot 2^k) \)

Statistics: 461 submissions, 40 accepted, first after 00:17
## Problem
Dynamically keep track of “uniqueness values” of cards while cards are being sold off.

## Solution
- When card is sold, at most 6 other cards (the 2 adjacent cards of each color) can change their uniqueness values.
- For $c \in \{R, G, B\}$, keep set $S_c$ of cards ordered by angle in color $c$.
- When selling card, find the $\leq 6$ affected cards and recompute their uniqueness values, using fast lookup in the sets $S_c$.
- Keep all cards in another ordered set ordered by uniqueness value for fast extraction of next card to sell.

Complexity is $O(n \log n)$ with balanced search trees or similar.
## Problem

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Complexity is \( O(n \log n) \) with balanced search trees or similar.

Statistics: 105 submissions, 14 accepted, first after 01:10
Problem
Assign pool of weak/normal/strong people to 2-person kayaks (of different speed factors) to maximize speed of slowest kayak.

Solution

1. Binary search over the answer.
2. Check feasibility by greedily assigning people to kayaks requiring strong+strong or strong+normal get that kayaks that can handle weak+weak or weak+normal get that pair up remaining weak with strongs and normals with normals and check if this can make all kayaks fast enough.

Time complexity is $O(n \log n)$ for $n$ people.
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2. Check feasibility by greedily assigning people to kayaks
   - kayaks requiring strong+strong or strong+normal get that
   - kayaks that can handle weak+weak or weak+normal get that
   - pair up remaining weak with strong and normal with normal and check if this can make all kayaks fast enough

Statistics: 82 submissions, 23 accepted, first after 00:46

Problem Author: Ulf Lundström  NCPC 2017 solutions
**Problem**

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Time complexity is \( O(n \log n) \) for \( n \) people.
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Problem
Find fastest way of walking distance \( L \). At certain points \( x_i \) we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

Solution
1. Let \( S(i) \) be best time if we start from \( i \)'th cart.
Problem

Find fastest way of walking distance $L$. At certain points $x_i$ we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

Solution

1. Let $S(i)$ be best time if we start from $i$’th cart.
2. Easy dynamic programming: for each $j > i$, try buying next coffee at cart $j$

$$S(i) = \min_{j > i} S(j) + \text{Time to go from } x_i \text{ to } x_j$$
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3. Alas, leads to $\Omega(n^2)$ time – too slow!
Problem

Find fastest way of walking distance $L$. At certain points $x_i$ we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

Solution

From each $x_i$, three categories of choice for best next cart $x_j$: 
During cooldown, during drinking, and after finishing the coffee.
A — Airport Coffee

Problem

Find fastest way of walking distance $L$. At certain points $x_i$ we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

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2. Before/After drinking: best to pick smallest such $j$ (get next coffee as soon as possible)
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2. Before/After drinking: best to pick smallest such $j$ (get next coffee as soon as possible)

3. During drinking: best to pick largest such $j$ (keep drinking coffee as long as possible)
Problem

Find fastest way of walking distance $L$. At certain points $x_i$ we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

Solution

1. Only need to consider three values of $j$ when computing $S(i)$. 

Overall complexity $O(n \log n)$. (Exercise: can be improved to $O(n)$ time.)
Problem

Find fastest way of walking distance $L$. At certain points $x_i$ we can choose to get a future temporary speedup (by buying a coffee), at the cost of cancelling any current speedup.

Solution

1. Only need to consider three values of $j$ when computing $S(i)$.
2. Can use binary search over cart positions to find them in $O(\log n)$ time.
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Overall complexity $O(n \log n)$.
(Exercise: can be improved to $O(n)$ time.)

Statistics: 65 submissions, 7 accepted, first after 02:50
Problem

Given a set of rays from in 2D (0, 0), and some points, assign max #points to a ray of minimum angular distance from the point, subject to ray capacities.

Phase 1

1. Sort all rays and points by angle.
2. For each point, compare distances to its two neighboring rays (Use sweep approach or binary search to find the two rays quickly).
3. Caveat! If using doubles, need to be careful with $\epsilon$. (Turns out, distances can differ by $\approx 10^{-13}$ without being equal.)
4. Can also check this using only integer computations. (But, despite small coordinates, need 64 bits.)
5. Points with a unique neighboring ray can be immediately assigned to that ray (if it has capacity left).
### Problem

Given a set of rays from in 2D \((0,0)\), and some points, assign max #points to a ray of minimum angular distance from the point, subject to ray capacities.

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Phase 2

1. For remaining points (having two closest rays) we get a bipartite matching problem between points and rays.
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Given a set of rays from in 2D \((0,0)\), and some points, assign max #points to a ray of minimum angular distance from the point, subject to ray capacities.

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1. For remaining points (having two closest rays) we get a bipartite matching problem between points and rays.
2. Graph is very simple (either a cycle, or a collection of paths)
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2. Graph is very simple (either a cycle, or a collection of paths)
3. Approach 1: solve using max flow (merging all points with the same angle into a single node with capacity = #points)
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   1. Time complexity with Ford-Fulkerson is $O(p^2)$ where $p$ is the number of train lines adjacent to some remaining person.
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   1. Time complexity with Ford-Fulkerson is $O(p^2)$ where $p$ is the number of train lines adjacent to some remaining person.
   2. However $p$ is hard to analyze. Turns out that $p \approx \max \text{coordinate} = 1000$, so this approach is fast enough.
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   1. First cut the cycle anywhere to get a path, solve path with simple greedy
Problem

Given a set of rays from in 2D \((0, 0)\), and some points, assign max #points to a ray of minimum angular distance from the point, subject to ray capacities.

Phase 2

1. For remaining points (having two closest rays) we get a bipartite matching problem between points and rays.
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Time complexity is \(O(n \log n)\).

Statistics: 17 submissions, 0 accepted

Problem Author: Johan Sannemo

NCPC 2017 solutions
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**Problem**

Given a huge tree with potentially $100000^{2^{30}}$ vertices, find distances between pairs of vertices.

**Solution**

1. A copy of $F_{30}$ will have at least $2^{30}$ vertices (assuming $F_0$ has at least 2 leaves).
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2. If $k > 30$, only relevant part is bottom-left copy of $F_{30}$ and the path to this subtree.
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Solution

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2. If $k > 30$, only relevant part is bottom-left copy of $F_{30}$ and the path to this subtree.
3. Reduces the problem to $k \leq 30$. 
**Problem**

Given a huge tree with potentially $100000^{2^{30}}$ vertices, find distances between pairs of vertices.

**Solution**

![Representation of vertex a in the big tree:](image)
Problem

Given a huge tree with potentially $100000^{2^{30}}$ vertices, find distances between pairs of vertices.

Solution

Representation of vertex $a$ in the big tree:

1. Record sequence $(a_1, a_2, \ldots)$ of leaves picked in each copy of $F_0$ when going from root to $a$
Problem
Given a huge tree with potentially $100000^{2^{30}}$ vertices, find distances between pairs of vertices.

Solution

$F_0$

\[
 a = (5, 2,  \ldots )
\]

Representation of vertex $a$ in the big tree:

Record sequence $(a_1, a_2, \ldots)$ of leaves picked in each copy of $F_0$ when going from root to $a$
Problem

Given a huge tree with potentially $100000^{230}$ vertices, find distances between pairs of vertices.

Solution

$F_0$

$F_2$

$1$

$2$

$3$

$4$

$5$

$a = (5, 2, 3)$

1. Representation of vertex $a$ in the big tree:
   1. Record sequence $(a_1, a_2, \ldots)$ of leaves picked in each copy of $F_0$ when going from root to $a$
   2. Finally add which node $a$ corresponds to in last copy of $F_0$. 
**Problem**

Given a huge tree with potentially $100000^{230}$ vertices, find distances between pairs of vertices.

**Solution**

1. Representation of vertex $a$ in the big tree.

2. Can find this representation in $O(k \log n)$ time using binary search and precomputation of subtree sizes.
F — Fractal Tree

Problem

Given a huge tree with potentially \(100000^{2^{30}}\) vertices, find distances between pairs of vertices.

Solution

1. What is distance between \((a_1, \ldots, a_p)\) and \((b_1, \ldots, b_q)\)?

\[d(a_1, b_1) = \text{distance between } a_1 \text{ and } b_1 \text{ in } F_0 \text{ (can compute it using a lowest common ancestor (LCA) algorithm)}\]

\[h(x) = \text{depth of node } x \text{ in } F_0\]
Problem

Given a huge tree with potentially $100000^{2^{30}}$ vertices, find distances between pairs of vertices.

Solution

1. What is distance between $(a_1, \ldots, a_p)$ and $(b_1, \ldots, b_q)$?
2. First remove any common prefix
   (moving into the same subtree does not affect distance)
Problem

Given a huge tree with potentially $100000^{2^{30}}$ vertices, find distances between pairs of vertices.

Solution

1. What is distance between $(a_1, \ldots, a_p)$ and $(b_1, \ldots, b_q)$?

2. First remove any common prefix
   (moving into the same subtree does not affect distance)

3. Then, when $a_1 \neq b_1$, distance is
   \[
   d(a_1, b_1) + \sum_{i=2}^{p} h(a_i) + \sum_{i=2}^{q} h(b_i)
   \]

4. $d(a_1, b_1) =$ distance between $a_1$ and $b_1$ in $F_0$
   (can compute it using a lowest common ancestor (LCA) algorithm)

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Statistics: 5 submissions, 0 accepted
253 teams
611 contestants
2819 total number of submissions
10 programming languages used by teams

Ordered by #submissions: C++ (1016), Java (865), Python (763), C (67), C# (65), Haskell (16), Prolog (9), Scala (8), Go (6), Ruby (4)

438 number of lines of code used in total by the shortest jury solutions to solve the entire problem set. (Significantly smaller than previous years – no killer problem in terms of implementation this year.)
Random facts

- All but two of the problems have **near-linear solutions**

  Exceptions:
Random facts

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  Exceptions:
  - D (Distinctive Character) – $O(n + k \cdot 2^k)$ solution
  - I (Import Spaghetti) – $O(n^3)$ solution.
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- The jury wrote Python solutions for all problems except C (Compass Card Sales). But mostly in Python 2, which is faster than Python 3 on Kattis due to using pypy instead of CPython. The Python solutions are always the shortest (often by a wide margin).
What now?


- Each university sends up to two teams to NWERC to fight for spot in World Finals (April 2018, in Beijing, China)